

Leader Notes: Flying Off the Handle

Explore/Explain Cycle I

Purpose:

Investigate generating and solving systems of equations. Use graphing calculator technology to generate a quadratic function by solving a system of equations. Apply this quadratic function to solve a problem.

Descriptor:

When a marble rolls down a ramp then off the edge, it will exhibit projectile motion until it reaches the ground. Participants will overlay a coordinate system to this problem situation. By finding three data points (coordinates of the point where the marble leaves the ramp, coordinates of the point where the marble hits the ground, and coordinates of the point where the marble hits a chair or desk placed in its path), participants will generate a quadratic function. They will use this quadratic function to predict where they need to place a cup so that the marble will land inside the cup.

Duration:

2 hours

TEKS:

- a(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.
- a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.
- 2A.1(B) Collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.
- 2A.3(A) Analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems.
- 2A.3 (B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.
- 2A.3 (C) Interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

- 2A.4(A) Identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).
- 2A.4(B) Extend parent functions with parameters such as a in $f(x) = \frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.
- 2A.6(B) Relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions.
- 2A.6(C) Determine a quadratic function from its roots or a graph.
- 2A.8(A) Analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems.
- 2A.8(C) Compare and translate between algebraic and graphical solutions of quadratic equations.
- 2A.8(D) Solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

TAKS Objectives Addressed by these Algebra 2 TEKS:

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 10: Mathematical Processes and Mathematical Tools

Technology:

- Graphing Calculator

Materials:

Advanced Preparation: The ramp should be constructed before the presentation (see directions following); Transparencies 1 and 2

Presenter Materials: projector (computer or overhead) for graphing calculator

Per group: ramp, marble, tape measure, desk or chair, carbon paper (or NCR form), one sheet of copy paper, hard flat plastic surface (for carpeted rooms only), tape, one cup, 2 or 3 textbooks

Per participant: graphing calculator, activity sheets

Leader Notes:

Graphing calculators should be a part of the high school mathematics classroom culture in Texas since the Texas Essential Knowledge and Skills for every high school mathematics course

require students to “use technology...to model mathematical situations to solve meaningful problems.” Furthermore, testing regulations for the Texas Assessment of Knowledge and Skills require students to have access to graphing calculators during the test. In this lesson, participants will collect data manually then use a graphing calculator to solve a meaningful problem.

Marble Ramp Construction

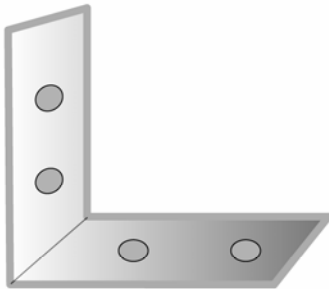
1 - piece of 2 x 2 x 8' wood (See Figure 1)

1 - 8' piece of corner molding (will actually use ~5')

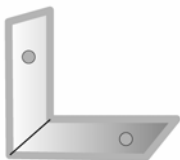
2 - straight 3" connectors



4 - 90° 3" brackets to use as legs



1 - 90° bracket (1 1/2" x 1 1/2")



Instructions:

1. Gather the materials shown above.
2. Cut the 2 x 2 piece of wood into three sections cutting 45° angles as shown.

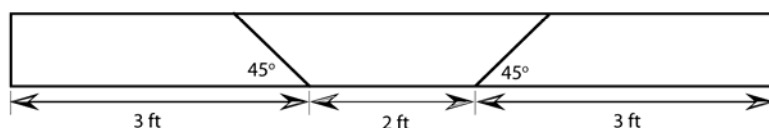


Figure 1

3. Make a groove in the 2 x 2 to hold the molding. (optional)

18 - wood screws (1/2" - #10)



4 - wood screws (2" - #10)



10 - small brads (1/2")



wood filler (optional)

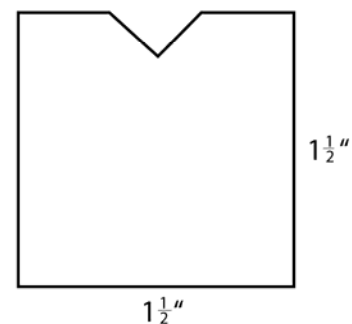
hammer

Phillips head screwdriver

saw

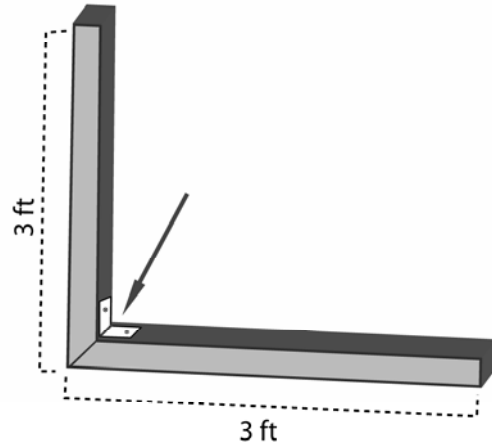
wood glue

miter box

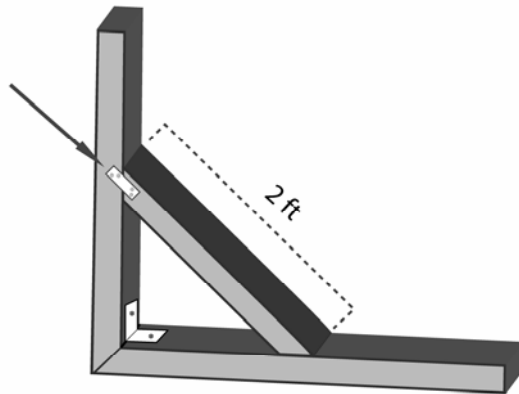


Note: Actual dimensions of 2 x 2 with optional notch

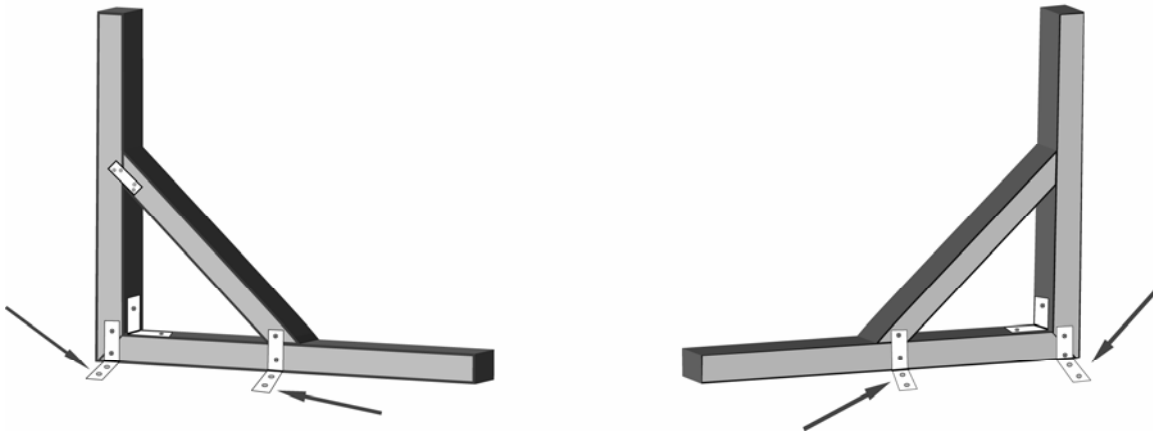
4. Join the two 3 ft pieces of wood together to form an L-shaped frame using the $\frac{1}{2}$ " screws and the 90° bracket ($1\frac{1}{2}$ " x $1\frac{1}{2}$ ").



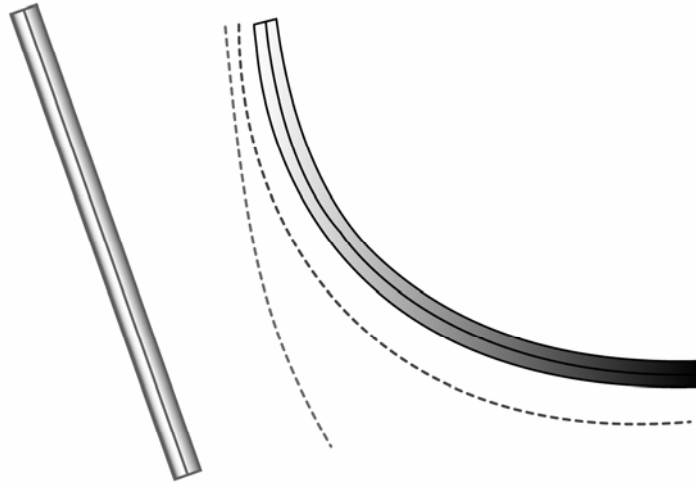
5. Attach the 2 ft piece of wood to the L-shaped frame with the 3" straight connectors and $\frac{1}{2}$ " screws at the connection point on the frame on both sides as shown.



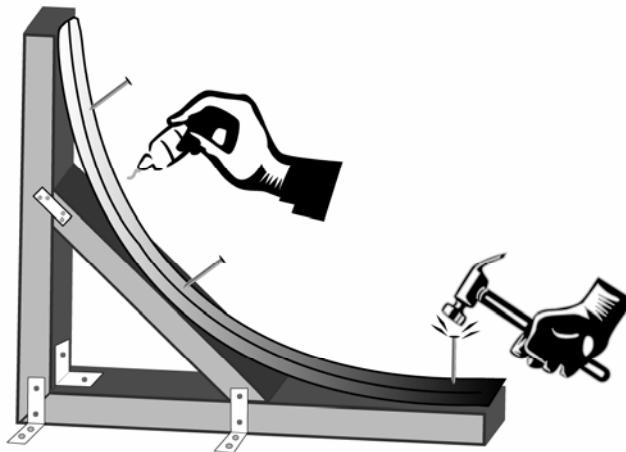
6. Attach the four 90° 3" brackets (legs) at the four positions on the frame with $\frac{1}{2}$ " screws.



7. Bend the flexible corner molding so that it is in a curved shape.

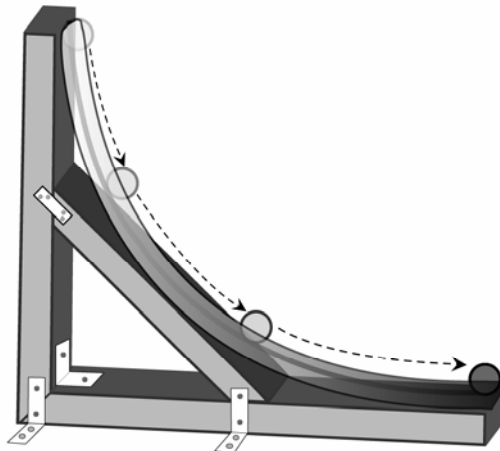


8. Glue and nail the molding to the frame using a nail set.



9. Fill in any nail holes with wood filler and sand as necessary. (*optional*)

10. The finished product will resemble the figure below and the marble will move as illustrated.



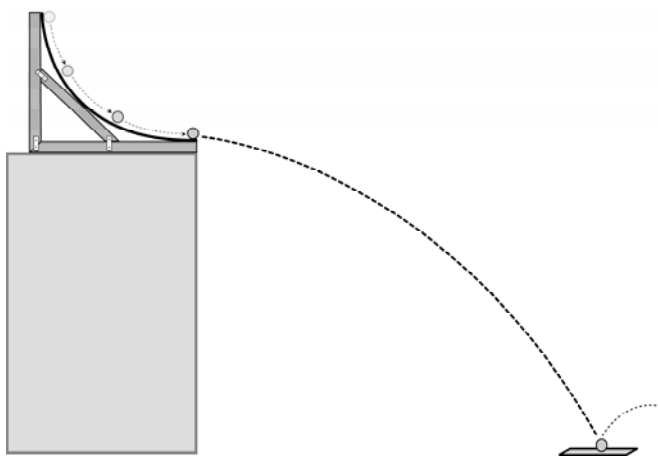
Explore

Posing the Problem:

In Hollywood movies, a classic way to end a car chase is to have a car drive off a cliff or into a canyon, with the hero jumping out of the car at the last minute to safety. Movie directors need to know exactly where the car will land so that they can have cameras in place to capture the motion on film. They also need to know where the car will be as it falls from the edge of the cliff to the floor of the canyon below so that they can have cameras in place there, too. How can we model this motion? How can we harness technology to apply this model to pinpoint the specific location of a moving object at any time?

To answer these questions, let's build a model that will enable us to simulate a car driving off a cliff. Use a wooden ramp and marble to simulate the car's motion. How would we develop a function rule to determine the placement of the cup on top of a stack of textbooks?

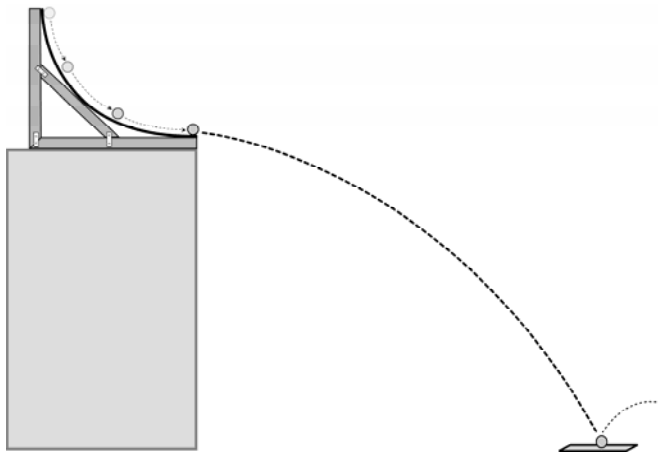
Note to Leader: when setting up the exploration, be sure to stress to participants that, as with producing movies, they will only get one shot to land the marble inside the cup. Hence, it is important that they collect data and build an accurate model.



Obtaining and Analyzing the Data:

Note to Leader: Set up the ramp on a high table. Roll the marble down the ramp and let it hit the floor to demonstrate the motion of the marble.

Display Transparency 1: Ramp Setup.



1. Let the floor represent the x -axis and the end of the ramp be contained on the y -axis. Where would the origin of this coordinate system be?

The origin is the point on the floor directly beneath the end of the ramp.

2. In this coordinate system, what do x and y represent?

x represents the horizontal distance from the end of the ramp and y represents the height above the floor.

Facilitation Questions

- Is there a dependency relationship between the x and y variables?
No. The variables are related, but there is not a clear dependency between x and y .
- What is the relationship between the x and y variables?
Both x and y represent distances that are dependent on the time that has elapsed since the marble left the end of the ramp.

3. Consider the path of the marble. Based on your coordinate system, what does the y -intercept represent? What are the coordinates of the y -intercept? Record the coordinates as a point in the table.

The y -intercept is the point at the end of the ramp. Its x -coordinate is 0 and its y -coordinate is the height of the end of the ramp above the floor. The coordinates are recorded in the table shown with Question 6.

Facilitation Question

- Does the precision of measurement matter?
Yes. The greater the precision of measurement of the distances, the greater the accuracy of the model.

4. Based on your coordinate system, what does the x -intercept represent?

The x -intercept is the point where the marble lands on the floor.

Roll the marble down the ramp and notice where it strikes the floor. Tape a piece of carbon paper on top of a piece of typing paper (carbon side down) where the marble touched the floor. Tape the paper to the floor.

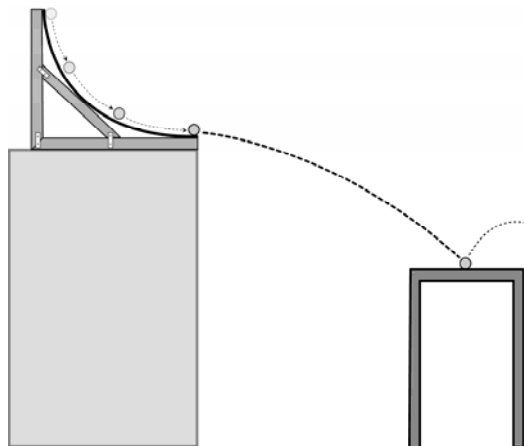
Note to Leader: If you are working in a room with carpeted floors, you will need to have a hard plastic or wooden surface to put under the paper. Otherwise, the impact of the marble will be dampened by the carpet and the marble will not leave a mark on the paper.

Roll the marble down the ramp and let it strike the paper on the floor. Repeat at least twice so that you have data for at least 3 trials.

Facilitation Questions

- Does the marble always land in the same spot? Why or why not?
Typically, no. Many factors can contribute to the motion of the marble, including friction with the ramp, the wobbling motion of the marble while falling down the ramp, or the curvature of the ramp itself.
- How can you determine the “average” location where the marble hits the floor?
Answers may vary. Participants should find a middle value that represents where the marble ought to reach the floor, allowing for some variance in location. Some participants may measure the three distances then find the arithmetic mean.

5. **What are the coordinates of the x -intercept? Record the coordinates as a point in the table.**
6. **Place a chair or desk between the ramp and the point of impact on the floor. Repeat your data collection procedure to find the x - and y -coordinates of the point of impact on the chair or desk. Record your third data point in the table.**
Display Transparency 2: Collecting the Third Data Point.



Sample Answers (in inches):

Horizontal Distance (x)	Height of the Marble (y)
0	65.5
58.75	0
34	39

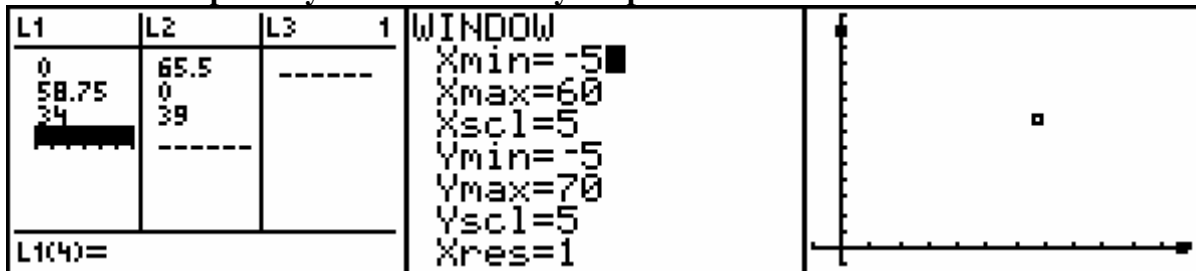
Facilitation Questions

- Where will the marble land on the chair or desk?
Roll the marble down the ramp to observe where the marble lands.
- Where should you place the carbon paper?
Place the carbon paper where the marble lands on the chair or desk.
- In your coordinate system, what would the x -value represent?
The x -value is the horizontal distance from the end of the ramp to the point where the marble lands on the chair or desk.
- In your coordinate system, what would the y -value represent?
The y -value is the vertical distance between the point where the marble lands on the chair or desk and the floor.

7. **What kind of functional relationship do you think exists between the horizontal distance and the vertical distance of the marble?**

Participants should predict a quadratic relationship due to the nature of free-fall and projectile motion.

8. **Make a scatterplot of your data. Sketch your plot.**



9. **Use the coordinates of the three data points to write a function rule that could be used to predict the height of the marble, y , when it is a horizontal distance, x , from the ramp. Explain how you found your function.**

The quadratic function modeling the sample data is $y = -0.0136x^2 - 0.3185x + 65.5$.

Methods of finding the function will vary. Participants could write a system of equations in standard form then use matrices to solve the system. Participants could also use transformations on the parent function $y = x^2$ in order to fit a curve to the data. Quadratic regression could also be used to find a function rule, depending on the nature of the course.

Facilitation Questions

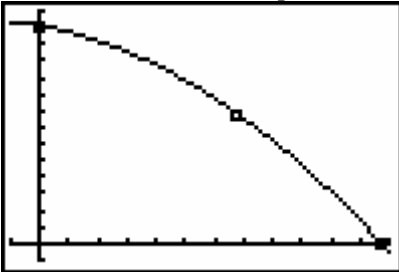
- Based on your answer to Question 7, what is the standard form for that type of function?
The standard form for a quadratic equation is $ax^2 + bx + c = 0$.
- How could you set up a system of equations to solve for the parameters of your standard form equation?
Substitute the known x - and y -values for each ordered pair and simplify.
- How could you solve this system of equations?
Answers may vary. Since the numbers are rather unwieldy, matrices could be a good tool to solve this system of equations.

Facilitation Questions, continued

- Does the graph of the parent function model the data well?
No.
- What transformations could you do to the parent function to obtain a model that fits the data well?
A vertical reflection across the x-axis, a vertical shift of b units (b represents the y-coordinate of the y-intercept), and a vertical compression
- How could you use technology to make solving the problem easier?
A graphing calculator can do matrix operations. Also, the graphing calculator and Excel will use quadratic regression features to generate a quadratic model for the data.

- 10. Graph your function rule over your scatterplot and sketch your graph. Is the function rule a good fit? How can you tell? If not, how can you revise your function rule so that it is a better fit?**

Answer based on sample data:

**Facilitation Question**

- What transformations could you do to your model to obtain a model that better fits the data?
A vertical reflection across the x-axis, a vertical shift of b units (b represents the y-coordinate of the y-intercept), and a vertical compression.

- 11. Place your cup on top of three textbooks. Where do you need to place the cup so that the marble will roll off the ramp and land inside the cup? Justify your choice.**

Responses will vary depending on the size of the cup and textbooks. Participants should stack the textbooks and place the cup on top then measure the height of the cup above the ground. Using this height, they should determine the horizontal distance from the end of the ramp to the center of the top of the cup.

Facilitation Questions

- What can you directly measure regarding the cup and textbooks?
The height of the cup and thickness of the textbooks can be measured.
- Where will the marble travel to land inside the cup?
The marble must clear the front lip of the cup, so aiming for the center of the cup will get the marble inside the cup.
- What are the coordinates of this part of the cup? How do you obtain them?
The coordinates are obtained by substituting the known y-value into the function rule and solving for x. This value will be the horizontal distance that the center of the cup will need to be from the edge of the ramp.

12. Test your prediction. Was your prediction correct? Why or why not? If not, revise your prediction and test it again.

Note to Leader: Weigh the cup down then pad the bottom of the cup with tissue or crumpled up napkins so that the marble lands inside the cup without bouncing out or knocking the cup over.

Facilitation Questions

- Should you move your cup toward or away from the ramp? Why?
Answers may vary. If the marble lands in front of the cup, the cup will need to be moved closer. If the marble lands behind the cup, the cup will need to be moved farther.
- Is your function model correct? Why or why not?
Answers may vary. The accuracy and precision of measurement of the original three points will greatly impact the accuracy of the function model.
- How can we generate a better function model?
Perform transformations on the model to yield a better fit. If this does not work, participants may need to recollect their three data points, paying attention to accuracy and precision of measurement, and generate a new function model based on their new data.

Explain

In this phase, use the debrief questions to prompt participant groups to share their responses to the data analysis. At this stage in the professional development, participants should be familiar with using the graphing calculator. Some participants may be familiar with using a spreadsheet such as Excel to analyze data.

1. How did you develop your function rule? Why did you choose this method?

Ask participants to discuss their methods and their reasons for making that choice. If none of the participants choose a particular method, ask participants why no one made that choice.

Using Matrices:

Beginning with the polynomial form of a quadratic function, $y = ax^2 + bx + c$, substitute values of x and y for the data points:

$$65.5 = a(0)^2 + b(0) + c$$

$$65.5 = c$$

$$0 = a(58.75)^2 + b(58.75) + c$$

$$0 = 3451.5625a + 58.75b + c$$

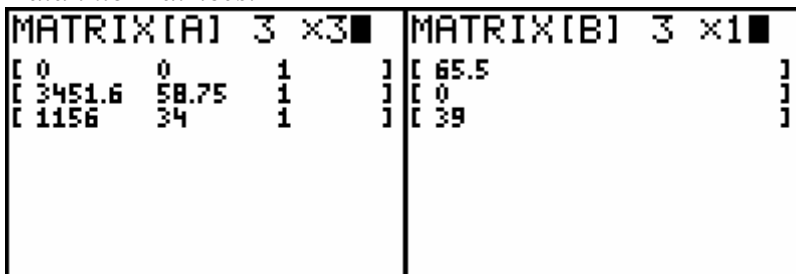
$$39 = a(34)^2 + b(34) + c$$

$$39 = 1156a + 34b + c$$

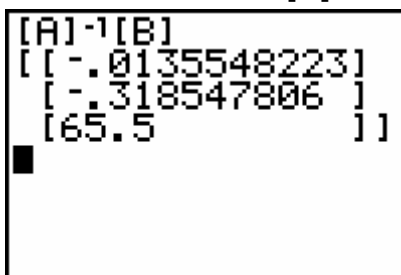
Write a matrix equation to represent this system.

$$\begin{bmatrix} 0 & 0 & 1 \\ 3451.5625 & 58.75 & 1 \\ 1156 & 34 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 65.5 \\ 0 \\ 39 \end{bmatrix}$$

Enter the coefficient matrix into the calculator's matrix A and the column matrix of the known terms into matrix B. For detailed instructions, see "Technology Tutorial: Entering Data into Matrices."



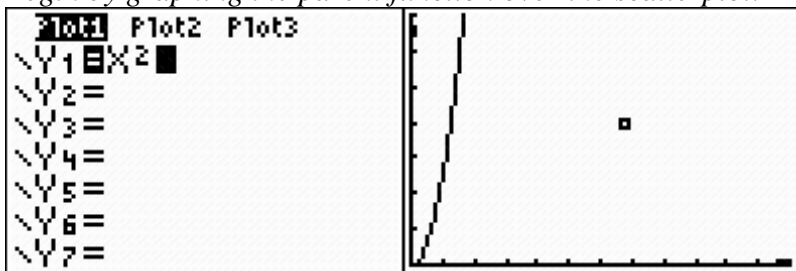
To solve the system, we need to left-multiply the matrix equation by the inverse of the coefficient matrix, or $[A]^{-1}$:



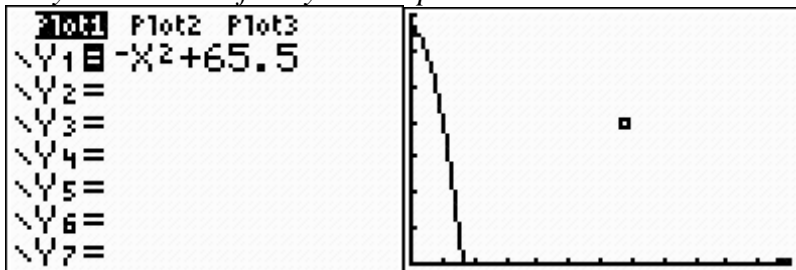
Thus, the quadratic function modeling our data is $y = -0.0136x^2 - 0.3185x + 65.5$.

Using Transformations:

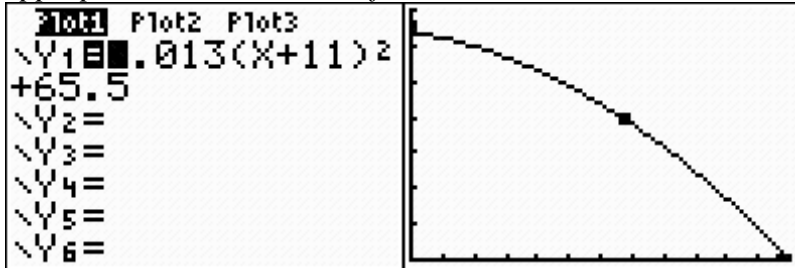
Begin by graphing the parent function over the scatterplot.



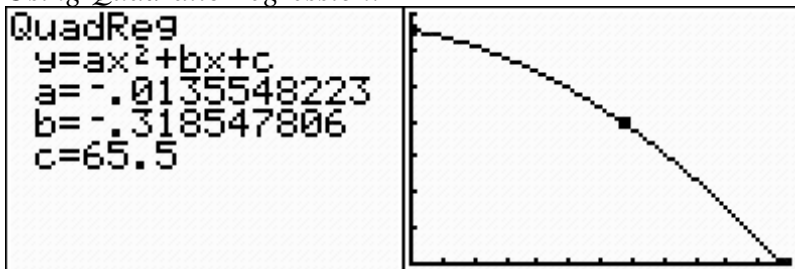
Reflect the parent function over the x-axis then vertically shift the parabola by the value of the y-coordinate of the y-intercept.



Continue using transformations, including vertical stretches or compressions, until an appropriate model has been found.



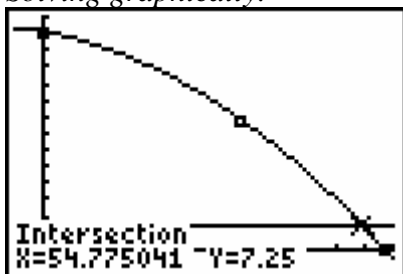
Using Quadratic Regression:



2. How did you determine the location for your cup? Why did you choose this method?

Responses will vary depending on the height of the cup and the thickness of the textbooks. In this example, assume that the textbooks are 1 inch thick each and that the cup is 4.25 inches tall. This assumption gives a total height of $1 + 1 + 1 + 4.25 = 7.25$ inches. Thus, we need to find the horizontal distance when the marble is 7.25 inches above the ground; i.e., we need to solve for x when $y = 7.25$.

Solving graphically:



Solving Tabularly:

X	Y1
54	8.6434
54.5	7.7464
55	6.8425
55.5	5.9319
56	5.0144
56.5	4.0902
57	3.1591

X=54.5

The cup needs to be placed so that the center of the cup is about $54\frac{3}{4}$ inches from the foot of the ramp.

3. How accurate were your predictions? If they needed revision, how did you decide on the revisions?

Responses will vary. If a cup has a small opening and participants are not careful of their precision of measurement, they may need to re-measure their data points to get a more accurate model. Some participants may report that they incorrectly solved the equation and had to solve the equation using another method.

4. What problem-solving strategies did you use to solve this problem?

Answers may vary. Possible answers may include “solving a simpler problem,” “using a model,” or “writing an equation.”

5. Could you use a technology other than the graphing calculator to solve this problem?

Answers may vary. See “Technology Tutorial: Flying Off the Handle” for details.

Note to Leader: Record or have a participant volunteer record the responses to Questions 6 and 7 on chart paper to use in the Elaborate phase of the professional development.

6. What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?

Responses may vary.

The matrix operations on the calculator make it easy to solve a matrix equation. The graphical analysis features of the calculator make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.

However, the small screen is difficult to see, and the axes in the window cannot be labeled.

7. What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?

Responses may vary.

The regression equation is calculated quickly on the spreadsheet. The axes can be clearly labeled with numbers and text labels. Labeled axes help the participant to use the graph to estimate solutions to problems that can be solved graphically. The graph can be enlarged or reduced then copied and pasted into other computer documents such as a Word or PowerPoint document to communicate the solution to a problem.

However, the participant is limited to the regression equations available in the spreadsheet. There are no graphical analysis features in most spreadsheets, so only estimates rather than exact solutions can be obtained graphically.

8. What TEKS does this activity address?

Participants should brainstorm a list of TEKS that they believe they have covered in this activity. The Leader Notes contain a comprehensive list of the TEKS addressed in this phase of the professional development. If participants do not mention some of these TEKS, then ask them how the activity also covers them.

9. How does the technology that you used enhance the teaching of those TEKS?

Responses may vary. However, participants should note that using technology enables them to explore a mathematical concept to a much deeper level. In this case, using technology made solving the problem significantly easier than parallel methods using pencil and paper. Technology makes rich mathematics accessible to a variety of learning styles.

Flying Off the Handle: Intentional Use of Data

1. At the close of *Flying Off the Handle*, distribute the **Intentional Use of Data** activity sheet to each participant.
2. Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Also prompt the participants to identify two key questions that are emphasized during this activity. Allow four minutes for discussion.

Facilitation Questions

- Which mathematics TEKS form the primary focus of this activity?
- What additional mathematics TEKS support the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

3. As a whole group, discuss responses for two to three minutes.
4. As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Explore/Explain cycle. Allow three minutes for discussion.

Facilitation Question

- What attributes of the activity support the level of rigor that you identified?

5. As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.
6. Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will be used during the Elaborate phase as prompts for generating attributes of judicious users of technology.

Facilitation Questions

- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per participant?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per small group of participants?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) for the entire group of participants?

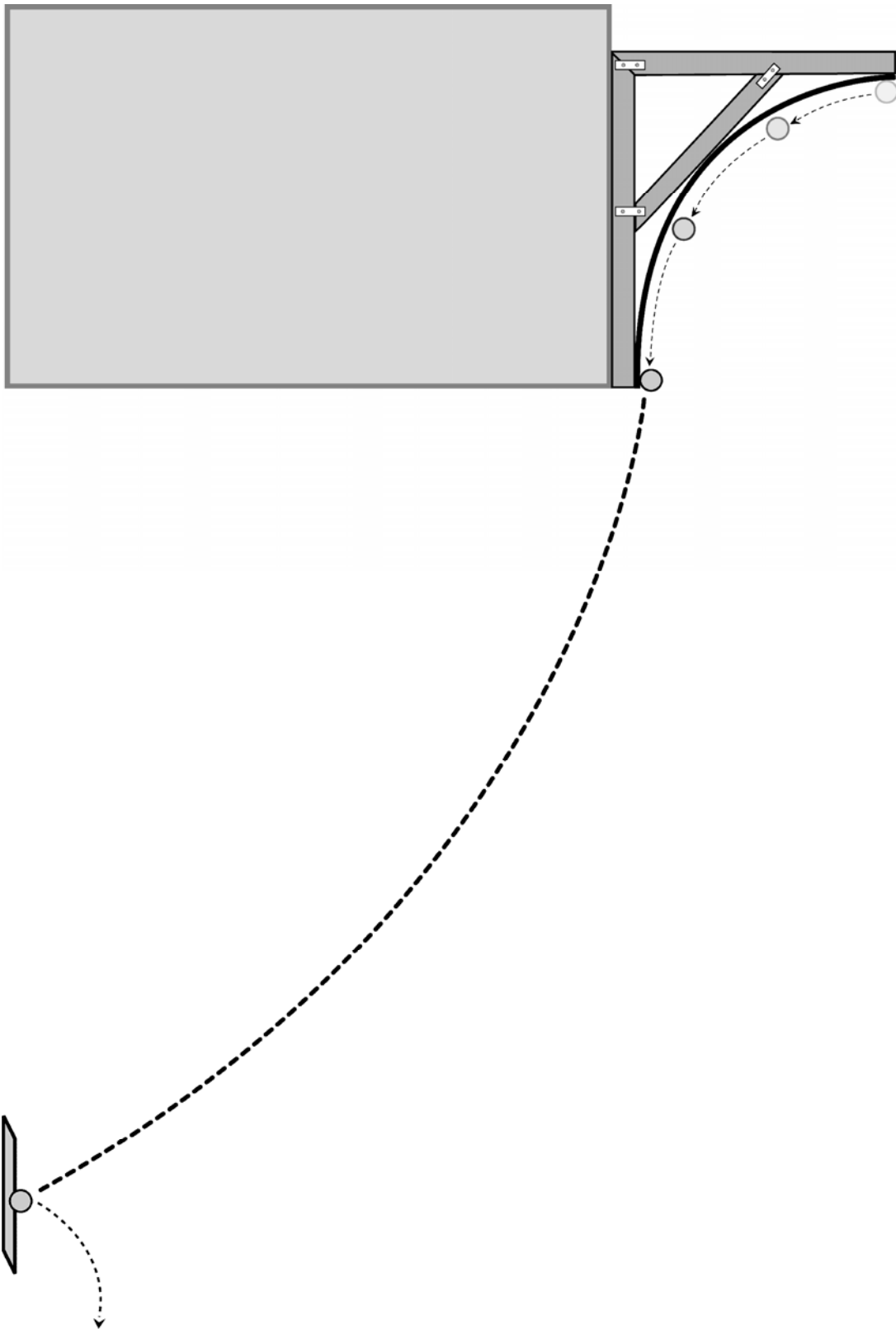
Facilitation Questions

- How would this activity change if we had used computer-based applications instead of graphing calculators?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

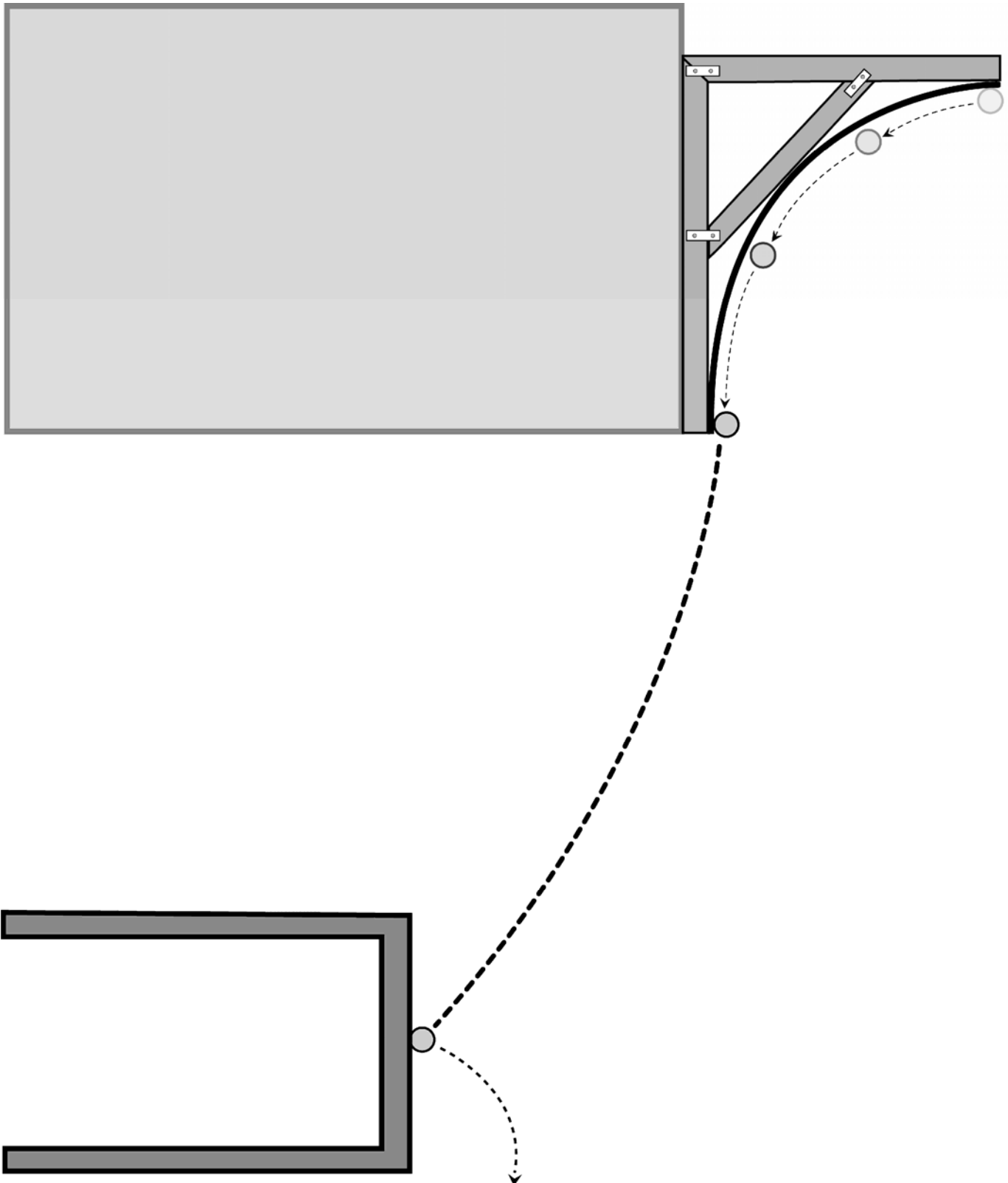
Sample Responses:

TEKS		<i>a(5), a(6), 2A.1B, 2A.3A, 2A.3B, 2A.3C, 2A.4A, 2A.4B, 2A.6B, 2A.6C, 2A.8A, 2A.8C, 2A.8D</i>	
Question(s) to Pose to Students	Math	<i>What type of relationship models the data you collected? How can you transform the parent function in order to better fit the data points?</i>	
	Tech	<i>How did technology help you with the analysis of data? How did technology help you to solve the problem?</i>	
Cognitive Rigor	Knowledge	√	
	Understanding	√	
	Application	√	
	Analysis	√	
	Evaluation	√	
	Creation	√	
Data Source(s)	Real-Time		
	Archival		
	Categorical		
	Numerical	<i>Three data points collected by measurement</i>	
Setting	Computer Lab		
	Mini-Lab		
	One Computer		
	Graphing Calculator	<i>Used to analyze the data, either via transformations, matrices to solve a system of equations, or quadratic regression</i>	
	Measurement-Based Data Collection	<i>Measured the distances using a meterstick or measuring tape</i>	
Bridge to the Classroom	<i>This activity could be done with Algebra 2 students as a motivating need to use matrices to solve systems of equations with matrices or to practice transforming the parent quadratic function. In Precalculus, this activity could be used with parametric equations, expressing the horizontal and vertical distances in terms of time.</i>		

Transparency 1: Ramp Setup



Transparency 2: Collecting the Third Data Point

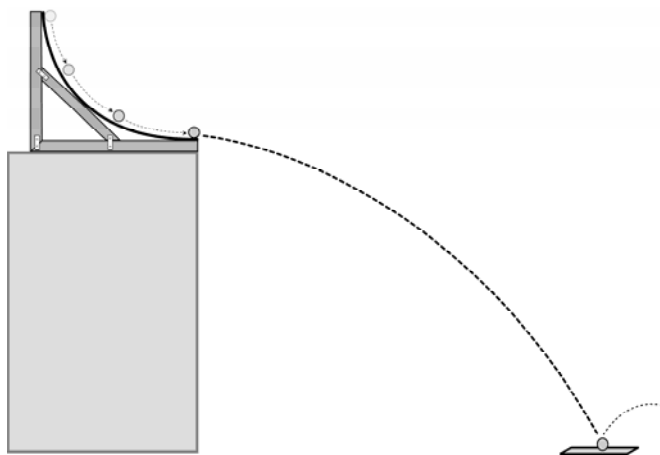


Flying Off the Handle

In Hollywood movies, a classic way to end a car chase is to have a car drive off a cliff or into a canyon, with the hero jumping out of the car at the last minute to safety. Movie directors need to know exactly where the car will land so that they can have cameras in place to capture the motion on film. They also need to know where the car will be as it falls from the edge of the cliff to the floor of the canyon below so that they can have cameras in place there, too. How can we model this motion? How can we harness technology to apply this model to pinpoint the specific location of a moving object at any time?

To answer these questions, let's build a model that will enable us to simulate a car driving off a cliff. Use a wooden ramp and marble to simulate the car's motion. How would we develop a function rule to determine the placement of the cup on top of a stack of textbooks?

Set up the ramp on a high table. Roll the marble down the ramp and let it hit the floor to observe the motion of the marble.



1. Let the floor represent the x -axis and the end of the ramp be contained on the y -axis. Where would the origin of this coordinate system be?
2. In this coordinate system, what do x and y represent?

3. Consider the path of the marble. Based on your coordinate system, what does the y -intercept represent? What are the coordinates of the y -intercept? Record the coordinates as a point in the table.

Horizontal Distance (x)	Height of the Marble (y)

4. Based on your coordinate system, what does the x -intercept represent?

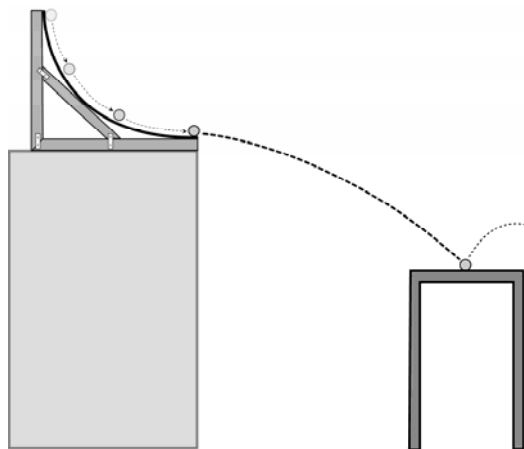
Roll the marble down the ramp and notice where it strikes the floor. Tape a piece of carbon paper on top of a piece of typing paper (carbon side down) where the marble touched the floor. Tape the paper to the floor.

Roll the marble down the ramp and let it strike the paper on the floor. Repeat at least twice so that you have data for at least 3 trials.

5. What are the coordinates of the x -intercept? Record the coordinates as a point in the table.

6. Place a chair or desk between the ramp and the point of impact on the floor. Repeat your data collection procedure to find the x - and y -coordinates of the point of impact on the chair or desk. Record your third data point in the table.

7. What kind of functional relationship do you think exists between the horizontal distance and the vertical distance of the marble?



11. Place your cup on top of three textbooks. Where do you need to place the cup so that the marble will roll off the ramp and land inside the cup? Justify your choice.
12. Test your prediction. Was your prediction correct? Why or why not? If not, revise your prediction and test it again.

Flying Off the Handle: Intentional Use of Data

TEKS		
Question(s) to Pose to Students	Math	
	Tech	
Cognitive Rigor	Knowledge	
	Understanding	
	Application	
	Analysis	
	Evaluation	
	Creation	
Data Source(s)	Real-Time	
	Archival	
	Categorical	
	Numerical	
Setting	Computer Lab	
	Mini-Lab	
	One Computer	
	Graphing Calculator	
	Measurement-Based Data Collection	
Bridge to the Classroom		